APPROXIMATE SOLUTIONS FOR A NEW VERSION OF THE CAR SEQUENCING PROBLEM (ABSTRACT)

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Abstract: This paper presents a solution procedure for a new variant of the Car Sequencing Problem (CSP), called xCSP. The aim is to satisfy the typical soft-constraints of the CSP while concentrating the maximum possible number of cars with specific options at specific times of the day in order to satisfy other production requirements. Additional constraint ratios are likewise considered that force at least a minimum specific number of consecutive options. The xCSP is formalized and computational results are presented proving the good performance of a GRASP procedure defined for it.

Key words: Meta-heuristic Search, Shop-Floor Scheduling, Car Sequencing Problem

1. INTRODUCTION

A variant of the sequencing problem of units in mixed product assembly lines is the so-called Car-Sequencing Problem (CSP). The problem, defined by Parello et al. (1986), consists in establishing the entry order onto an assembly line of a set of units (cars) of different types. Each unit of a specific type requires a subset of optionsthat normally require an extra application time at the dedicated workstation.

Each option is therefore subject to a set of maximum load constraint ratios of the form p_j/q_j, where p_j is the maximum number of times that option j may appear in each production sequencing segment of length q_j. In its original version, the CSP is a feasibility problem; i.e. given an instance of the problem, the aim is to find a sequencing of vehicles (if this exists) that satisfies all the maximum load constraints.

This problem, which has been shown to be NP-hard in the strong sense (Kis, 2004), has been mainly addressed by and received special attention from the scientific community devoted to Constraint Programming, CLP and has become the standard example of software demonstration in Constraint Satisfaction Problem solver manuals.

A natural extension of the CSP problem is proposed in the present paper. The aim of this extension is to: (1) incorporate into the obtained solutions certain properties that are desirable in real shops i for manufacturing in a JIT context (regularity in workloads and component consumption), and (2) penalize to a different degree the possible violation of constraints, which is not permitted in a feasibility problem.

On the one hand, a set of additional $r_j\backslash s_j$ minimum load constraints is incorporated into the original problem that represent the constraint that the minimum number of times that option j may appear in each production sequencing segment of length s_j is r_j. For example, in the binary version of CSP, a constraint of the type 1\S would indicate that, for such an option, there cannot be S consecutive vehicles without at least one of these taking this option.

This paper proposes, on the other hand, the conversion of the feasibility problem into an optimization problem by incorporating an objective function that will depend on the slackness variables of the maximum and minimum load constraints.

2. MODELING THE PROBLEM

Given an instance of the *Car-Sequencing Problem* ($C,P,D,d_i,c_{ji},p_j/q_j$), let us call *C and* the number of different possible options (air conditioning, sun-roof, etc); *P* the number of different car types (each with its own options); *D* (total demand) the total number of cars to sequence in the considered time horizon, d_i (partial demand) the number of cars of type i=1..P to sequence, $\Sigma d_i=D$; c_{ji} being the parameter that indicates the number of units of option j=1..C required by a car of type i=1..P (in the binary version of the CSP, $c_{ji}\in\{0,1\}$), and p_j/q_j the constraint ratios that, for each option j=1..C, indicate how many times this option may appear at the most in any sequencing of q_j vehicles. he basic model for the CSP may be defined by means of expressions (1)-(4):

$$\sum_{i=1}^{P} x_{it} = 1 \qquad \forall t = 1..D \tag{1}$$

$$\sum_{t=1}^{D} x_{it} = d_i \qquad \forall i = 1..P \tag{2}$$

$$\sum_{i=1}^{P} \sum_{k=l_j(t)}^{t} c_{ji} x_{ik} \leq min\{t, p_j\} \qquad \forall j = 1..C\,; \forall t = 1..D; l_j(t) = max\{1, t+1-q_j\} \tag{3}$$

$$x_{it} \in \{0,1\} \qquad \forall i = 1..P\,; \forall t = 1..D \tag{4}$$

2.1 xCSP: The Extended CSP model

On certain occasions to add constraints that force the carrying out of a minimum number of operations until a certain instant. This gives rise to a new group of soft-constraints, which we represent as $r_j \backslash s_j$, and which indicate that for each sequencing of s_j vehicles, option j has to be present at least r_j times. These constraints may be established by means of expressions (5).

$$\sum_{i=1}^{P} \sum_{k=h_j(t)}^{t} c_{ji} x_{ik} \geq \min\{t, r_j\} \qquad \forall j = 1..C\,; \forall t = 1..D; h_j(t) = \max\{1, t+1-s_j\} \tag{5}$$

where hj(t)=max{1,t+1-sj} is the commencement instance of sequencing segment [hj(t),t], t=1..D, of length equal to or less than s_j.

We define variables:

$$y_{jt}(U) = \left[\sum_{k=l_j(t)}^{t} n_{j,u(k)} - \min\{t, p_j\} \right]^+ \qquad \forall j = 1..C\,; \forall t = 1..D; l_j(t) = \max\{1, t+1-q_j\} \tag{6}$$

$$z_{jt}(U) = \left[\min\{t, p_j\} - \sum_{k=l_j(t)}^{t} n_{j,u(k)} \right]^+ \qquad \forall j = 1..C\,; \forall t = 1..D; l_j(t) = \max\{1, t+1-q_j\} \tag{7}$$

$$v_{jt}(U) = \left[\sum_{k=h_j(t)}^{t} n_{j,u(k)} - \min\{t, r_j\} \right]^+ \qquad \forall j = 1..C\,; \forall t = 1..D; h_j(t) = \max\{1, t+1-s_j\} \tag{8}$$

$$w_{jt}(U) = \left[\min\{t, r_j\} - \sum_{k=h_j(t)}^{t} n_{j,u(k)} \right]^+ \qquad \forall j = 1..C\,; \forall t = 1..D; h_j(t) = \max\{1, t+1-s_j\} \tag{9}$$

Note that any sequencing U=u(1)..u(D) that presents $y_{jt}(U)=0$ for every option j=1..C and for every instant t=1..D of the sequencing, is the solution of the CSP in its original version.

Defining some weights α, β, γ, δ respectively for each of these new variables, we obtain the final model for xCSP:

$$\min \sum_{t=1}^{D} \sum_{j=1}^{C} \left(\alpha_{jt} y_{jt} + \beta_{jt} z_{jt} + \gamma_{jt} v_{jt} + \delta_{jt} w_{jt} \right) \tag{10}$$

s.t. equations (1), (2) and (4) and:

$$z_{jt} - y_{jt} + \sum_{i=1}^{P} \sum_{k=l_j(t)}^{t} c_{ji} x_{ik} = \min\{t, p_j\} \quad \forall j = 1..C; \forall t = 1..D; l_j(t) = \max\{1, t+1-q_j\} \tag{11}$$

$$w_{jt} - v_{jt} + \sum_{i=1}^{P} \sum_{k=h_j(t)}^{t} c_{ji} x_{ik} = \min\{t, r_j\} \quad \forall j = 1..C; \forall t = 1..D; h_j(t) = \max\{1, t+1-s_j\} \tag{12}$$

$$y_{jt}, z_{jt}, v_{jt}, w_{jt} \geq 0 \quad \forall j = 1..C; \forall t = 1..D \tag{13}$$

3. A GRASP APPROACH

The complexity of the xCSP model and the practical interest of its solution in a reasonable time on the part of car manufacturers suggest the definition of heuristic procedures that are capable of providing acceptably good solutions with a low computational effort. In this paper we define a GRASP metaheuristic. With the purpose of testing its capability to solve xCSP instances, we have tested the algorithm on 79 instances published on the public library CSPLib, once adapted to the new constraints of the xCSP.

An appropriate experimental framework was defined (considering different weights in eq. (10) and different lengths for the GRASP candidate list), and the corresponding experiments were carried out. After analyzing the results, we can state that our algorithm is able to solve hardly all the instances for the new problem, using very reasonable computational times (around one second of CPU).

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